

Note: 1. Attempt all Sections. If require any missing data; then choose suitably.

SECTION A

1. Attempt all questions in brief.

2 x 10 = 20

- Show that the vectors $(1, 3, 4)$, $(7, -2, 1)$, and $(2, 5, 6)$ of $V^3(F)$ are linearly dependent.
- Define rank and nullity.
- Prove that $(1, 1, 1)$, $(0, 1, -1)$ and $(0, 1, -1)$ generate $R^3(R)$.
- Define Dual space.
- Suppose x, y are vectors in an inner product space $V(K)$. Show that $x = y$ iff $\langle x, z \rangle = \langle y, z \rangle \quad \forall z \in V$.
- Let T be a linear map from a vector space V into V . Prove that $R(T) \cap N(T) = \{0\}$
- Show that the set $S\{(1,0,0), (1,1,0), (1,1,1)\}$ is basis set of $R^3(R)$. Find the co-ordinate vector of (a, b, c) .
- Show that the following mapping is linear, $F: R^3 \rightarrow R^2$ given by $F(x, y, z) = (z, x + y)$.
- If α and β are vectors in an inner product space, then show that $\|\alpha + \beta\|^2 + \|\alpha - \beta\|^2 = 2\|\alpha\|^2 + 2\|\beta\|^2$
- State Extension theorem.

SECTION B

2. Attempt any three of the following:

10 x 3 = 30

- Show that the union of two subspaces is a subspaces iff one is contained in the other.
- If V and U are the vector spaces over the field K . let V be of finite dimension. Let $T: V \rightarrow U$ be a linear map, then $\dim V = \dim R(T) + \dim N(T)$.
- If W_1 and W_2 are two subspaces of a finite dimensional vector space $V(F)$, then $\dim W_1 + \dim W_2 = \dim(W_1 + W_2) + \dim(W_1 \cap W_2)$.
- Define Annihilators and if W be a nonempty subset of a vector space $V(K)$, then show that $\dim W^0 + \dim W = \dim V$, where W^0 is the Annihilator of W .
- Define the orthogonal compliment X^\perp of a subspace X of a vector space V and prove that X and X^\perp are complementary subspaces.

SECTION C

3. Attempt any one part of the following:

10 x 1 = 10

- A non-empty subset of W of a vector space $V(F)$ is a subspace of $V(F)$ if and only if W is closed w. r. t. the operations on V .
- State and prove Cauchy-Schwarz's inequality.

4. Attempt any one part of the following:

10 x 1 = 10

- Define null space and if T is a linear transform on a vector space V to a vector space W , then show that null space $N(T)$ is the subspace of V .
- If T be a linear transformation on a vector space V to W , then show that vectors

$x_1, x_2, \dots, x_n \in V$ are linearly independent if, Tx_1, Tx_2, \dots, Tx_n are linearly independent.

5. Attempt any one part of the following: 10 x 1 = 10

- (a) If F be the linear operator on R^3 define by $F(x, y, z) = (2y + z, x - 4y, 3x)$, then verify that $[F]_{e'}[v]_{e'} = [F(v)]_{e'}$ for the basis $\{e'_1 = (1, 1, 1), e'_2 = (1, 1, 0), e'_3 = (1, 0, 0)\}$
- (b) (i) If A is a linear transformation on an n -dimensional vector space, then show that $\rho(A) = \rho(A')$.
(ii) Show that function 'f' defined on vectors $\alpha = (x_1, x_2)$ and $\beta = (y_1, y_2)$ in R^2 , is bilinear form: $f(\alpha, \beta) = x_1y_2 - x_2y_1$

6. Attempt any one part of the following: 10 x 1 = 10

- (a) (i) Prove that an orthogonal set of vector is linearly independent.
(ii) Show that the subset $\{(a, b, c) : a + b + c = 0\}$ of $R^3(R)$ is a subspace of $R^3(R)$.
- (b) (i) Show that every finite dimensional vector space is an inner product space.
(ii) If T is the linear operator on $V_2(C)$ defined by $T(1, 0) = (1 + i, 2)$ and $T(0, 1) = (i, i)$, C is the set of complex numbers, Find the matrix representation of T^* relative to standard ordered basis.

7. Attempt any one part of the following: 10 x 1 = 10

- (a) (i) Prove that two vectors α and β in a real inner product space $V(R)$ are orthogonal iff $\|\alpha + \beta\|^2 = \|\alpha\|^2 + \|\beta\|^2$
(ii) Show that the intersection of two subspaces W_1 and W_2 of a vector space $V(F)$ is also a subspace of $V(F)$.
- (b) (i) If λ is an eigen value of an invertible operator T , then λ^{-1} is an eigen value of T^{-1} .
(ii) Verify Cayley Hamilton theorem for the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$.