

M. TECH.

(SSEM -2) THEORY EXAMINATION 2017-18
ADVANCED MECHANICS OF SOLIDS

Time: 3 Hours

Total Marks: 70

Note: Attempt all Sections. If require any missing data; then choose suitably.

SECTION A

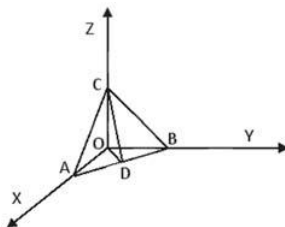
1. Attempt all questions in brief. 2 x 7 = 14

- What do you mean Cartesian tensor?
- Explain Stress Transformation?
- What is Octahedral strain?
- Give short note on hyperelastic material model?
- Write Nervier equation for boundary value problem?
- What do you mean by principle of virtual work?
- What do you mean by surface force?

SECTION B

2. Attempt any three of the following: 7 x 3 = 21

- If A be an arbitrary tensor. Show that $A^t A$ and AA^t are positive semi-definite symmetric tensors. If A is invertible, prove that these tensors $-A^t A$ and AA^t are positive definite.
- A cube of side 1 cm is subjected to a displacement field of the form, $u = (A * y + 2A * z)e_y + (3A * y - A * z)e_z$, where (x,y,z) denotes the coordinates of a typical material particle in the current configuration, $\{e_i\}$ the Cartesian coordinate basis and $A = 10^{-4}$, a constant. If the cube is made up of a material that obeys isotropic Hooke's law with Young's modulus, $E = 200$ GPa and Poisson's ratio, $\nu = 0.3$, (i) Find the Cartesian components of the Cauchy stress with respect to the $\{e_i\}$ basis. (ii) Identify whether this stress states correspond to the plane stress
- Consider the following homogeneous deformation: $x = a_1 X + k_1 Y$, $y = a_2 Y + k_2 Z$, $z = a_3 Z$, where a_i and k_i are constants and (X,Y,Z) are the Cartesian coordinates of a material particle in the reference configuration and (x,y,z) are the Cartesian coordinates of the same material particle in the current configuration. For this deformation field and the tetrahedron OABC shown in figure. such that $OA = OB = OC$ and $AD = DB$, compute (i) The change in length of the line segment AB (ii) The change in angle between line segment AB and CD (iii) The deformed surface area of the faces ABC, OAB and OBC.



- d. Define “Stress concentration” and explain the phenomenon with two examples.
- e. Discuss about the stress concentration at reentrant corners and explain the principle of membrane analogy and establish the analogous quantities.

SECTION C

3. Attempt any *one* part of the following: 7 x 1 = 7

- (a) Show that $y = mx$ where y and x are vectors and m is a scalar is a linear transformation. Find the second order tensor that maps the given vector x on to y .
- (b) Show that if tensor A is positive definite then $\det(A) > 0$

4. Attempt any *one* part of the following: 7 x 1 = 7

- (a) A cube of side 20 cm, is subjected to a uniform plane state of strain whose

$$\text{Cartesian components are: } \varepsilon = \begin{pmatrix} 1 & -2 & 0 \\ -2 & 3 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

For this constant strain field, find

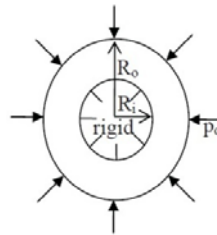
- (i) Maximum change in angle and the orientation of the material fibers along which this occurs (ii) Whether the deformation is isochoric.
- (b) State the Kirchhoff's Theorem for first and second system?

5. Attempt any *one* part of the following: 7 x 1 = 7

- (a) Show that a plane stress state will result in a plane strain state only when $\text{tr}(\sigma) = 0$ in a material that obeys isotropic Hooke's law. Recollect that this state of stress wherein $\text{tr}(\sigma) = 0$ is called pure shear state of stress.
- (b) Write a short note on the following a) Elastic constants b) Lamé's Theory

6. Attempt any *one* part of the following: 7 x 1 = 7

- (a) The annular cylinder has shown Fig. to be homogeneous and made of a material that obeys isotropic Hooke's law with Young's modulus, $E = 200$ GPa and Poisson's ratio, $\nu = 0.3$ (a) Obtain the location and magnitude of the maximum and minimum hoop stress and radial stress. (b) Obtain the maximum principal stress, its location and magnitude. Here the inner boundary is fixed $u_r(r_i, \theta, z)$. Find the hoop stress and radial stress as R_i/R_o tends to 0 and comment on the significance of this limit.



- (b) What are the basic Boundary Conditions for the surface of the body? Explain in brief.

7. Attempt any *one* part of the following: 7 x 1 = 7

- (a) Determine which of the following Cauchy stress fields are possible within a body at rest assuming that there are no body forces acting on it:

$$\sigma = \begin{pmatrix} 3x & -3x & 0 \\ -7y & 7y & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

- (b) What do you mean by total potential energy and complimentary potential energy?