

**B. TECH.
(SEM IV) THEORY EXAMINATION 2017-18
MATHEMATICS III**

Time: 3 Hours
Total Marks: 100
Note: Attempt all Sections. If require any missing data; then choose suitably.

SECTION A

1. Attempt all questions in brief.

2 x 10 = 20

- (a) Determine analytic function $f(z)$ in terms of z whose real part is $x^3 - 3xy^2$.
- (b) Write Cauchy's Reimann conditions in polar coordinates system
- (c) If the Fourier transform of e^{-x^2} is $\sqrt{\pi} e^{-p^2/4}$ then find Fourier transform of $e^{-5(x-2)^2}$.
- (d) Find Z transform of a^k .
- (e) Write normal equations to fit the curve $y = ax^2 + b$ by method of least square.
- (f) Find mean and variance of Poisson distribution.
- (g) Write the Newton Raphson iteration formula
- (h) Prove that third divided difference of $\left(\frac{1}{a}\right)$ is $-\frac{1}{abcd}$.
- (i) Discuss diagonal dominant property for system of linear equations.
- (j) Use Picard's method to obtain $y(0.2)$ up to two iterations. Given: $\frac{dy}{dx} = x - y$ with the condition $y(0) = 1$

SECTION B

2. Attempt any three of the following:

10 x 3 = 30

- (a) By contour integration find: $\int_0^\infty \frac{x \sin x}{x^2 + a^2} dx$; $a > 0$.
- (b) Using Z-transform solve the difference equation: $y_{k+2} + 4y_{k+1} + 3y_k = 3^k$, given $y_0 = 0, y_1 = 1$.
- (c) In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find mean and standard deviation of the distribution. It is given that $f(t) = \frac{1}{\sqrt{2\pi}} \int_0^t e^{-\frac{1}{2}x^2} dx$, then $f(0.5) = 0.19$ and $f(1.4) = 0.42$.
- (d) Find the real root of equation $3x + \sin x - e^x = 0$ by the method of False-position correct to three decimal places.

- (e) Find the value of $y(1.1)$ using Runge-Kutta method of fourth-order, given that $\frac{dy}{dx} = y^2 + xy$, $y(1) = 1$, take $h = 0.05$.

SECTION C

3. Attempt any one part of the following: **10 x 1 = 10**

- (a) State and prove Cauchy's Residue Theorem. Hence or otherwise evaluate $\oint_C \frac{z^2 - 2z}{(z+1)^2(z^2+4)} dz$, where C is $|z| = 10$.

- (b) Expand $\frac{7z-2}{z^3-z^2-2z}$ in the regions (i) $1 < |z+1| < 3$ (ii) $0 < |z+1| < 1$ (iii) $|z+1| > 3$

4. Attempt any one part of the following: **10 x 1 = 10**

- (a) Find Fourier cosine transform of $\frac{1}{1+x^2}$ and hence find Fourier sine transform of $\frac{x}{1+x^2}$

- (b) The temperature u in the semi-infinite rod $0 \leq x < \infty$ is determined by the differential equation $\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}$ subject to the conditions;

$$(i) u=0 \text{ when } t=0, x \geq 0 \quad (ii) \frac{\partial u}{\partial x} = -\mu$$

(a constant) when $x=0$ and $t>0$. Making use of cosine transform, show that

$$u(x, t) = \frac{2\mu}{\pi} \int_0^{\infty} \frac{\cos px}{p^2} (1 - e^{-kp^2 t}) dp$$

5. Attempt any one part of the following: **10 x 1 = 10**

- (a) The first four moments of a distribution about $x = 4$ are 1, 4, 10 and 45. Calculate the moments about the mean and comment upon the skewness and kurtosis of the distribution.

- (b) To test the effectiveness of inoculation against cholera, the following table was obtained:

	Attacked	Not attacked	Total
Inoculated	30	160	190
Not inoculated	140	460	600
Total	170	620	790

(The figures represent the number of persons)

Use χ^2 -test to defend or refute the statement that the inoculation prevents attack from cholera. ($\chi_{0.05}^2$ for 1 d.f. = 3.841).

6. Attempt any *one* part of the following: **10 x 1 = 10**

(a) By means of Newton's divided difference formula, find the value of $f(15)$ from the following table:

x	4	5	7	10	11	13
$f(x)$	48	100	294	900	1210	2028

(b) Estimate from the table, the number of students who obtained marks between 40 and 45.

Marks:	30-40	40-50	50-60	60-70	70-80
No. of students:	31	42	51	35	31

7. Attempt any *one* part of the following: **10 x 1 = 10**

(a) Solve by Crout's method, the following system of equations:

$$x + y + z = 3, \quad 2x - y + 3z = 16, \quad 3x + y - z = -3$$

(b) Find the approximate value of $\int_0^{\frac{\pi}{2}} \sin x \, dx$ by Simpson's rule.