Printed Pages : 3

EAS-103

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 199109

Roll No. | | | | | | | | | | |

B. Tech.

(SEM. I) (ODD SEM.) THEORY EXAMINATION, 2014-15

MATHEMATICS - I

Time : **3** Hours] [Total Marks : **100**

Note : Attempt **all** questions. All questions carry **equal** marks.

1 Attempt any **two** parts :

a) Find the n^{th} derivative of $\frac{(2x+1)}{(2x-1)(2x+3)}$.

b) If $z = \log(e^x + e^y)$ then show that $rt - s^{-2} = 0$ where $r = z_{xx}$, $t = z_{yy}$, $s = z_{xy}$. Symbols have their usual meanings.

c) Verify Euler's theorem for function $u = \log\left(\frac{x^4 + y^4}{x+y}\right)$.

2 Attempt any **TWO** parts of the following :

a) Expand $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ in ascending powers of x.

b) If u, v, w are roots of equation

$$(x-a)^3+(x-b)^3+(x-c)^3=0 \text{ find } \frac{\partial(u,v,w)}{\partial(a,b,c)}.$$

c) Using Lagrange's method of Maxima and Minima find the shortest distance from the point $(1,2,-1)$ to sphere $x^2+y^2+z^2=24$.

3 Attempt any **TWO** parts of following :

a) Find the rank of matrix by reducing to Normal form

$$A=\begin{bmatrix} 1 & 2 & 1 & 0 \\ -2 & 4 & 3 & 0 \\ 1 & 0 & 2 & -8 \end{bmatrix}.$$

b) Find the eigen values of corresponding eigen vectors of matrix $A=\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$.

c) If $A=\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, find A^{-1} and A^4 using Caylay's Hamilton's theorem.

4 Attempt any **TWO** parts of following :

a) Evaluate $\int_0^1\int_0^1 \sin y^2 dydx$ by changing the order of integration.

b) Evaluate $\int_0^\infty \int_0^\infty e^{-\left(x^2+y^2\right)} dxdy$ by changing into polar coordinates.

c) Prove that $\frac{\beta(p,q+1)}{q} = \frac{\beta(p+1,q)}{p} = \frac{\beta(p,q)}{p+q}$ $(p>0, q>0)$.

5 Attempt any **TWO** of following :

a) Using Green's theorem, evaluate $\int_c \left(x^2+xy\right)dx + \left(x^2+y^2\right)dy$ where c is square formed by lines $x=\pm 1$, $y=\pm 1$.

b) Verify divergence theorem for $\vec{F} = x^3\hat{i} - y^3\hat{j} + z^3\hat{k}$ taken over surface of sphere $x^2+y^2+z^2=a^2$.

c) If $\vec{F} = \left(x^2+yz\right)\hat{i} + \left(y^2+zx\right)\hat{j} + \left(z^2+xy\right)\hat{k}$ find $div\,\vec{F}$ and $curl\,\vec{F}$.
